



## Brief Communication

Investigation of peak frequencies in the flopping regime  
with a two-cylinder arrayD.W. Guillaume<sup>a,\*</sup>, J.C. LaRue<sup>b</sup><sup>a</sup> Department of Mechanical Engineering, California State University, Los Angeles, CA 90032, USA<sup>b</sup> Department of Mechanical and Aerospace Engineering, University of California, Irvine, CA 92697, USA

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**Abstract**

The variation of the base pressure coefficient ( $C_p$ ), velocity power spectra, and velocity time–frequency plots for an array of two cylinders aligned normal to the flow are presented. Power spectra of the fluctuating velocity downstream of the two-cylinder array with  $s/d = 0.750$  (where  $s$  is the spacing between the top and bottom surfaces of adjacent cylinders and  $d$  is the diameter of the cylinder) show two peaks. The corresponding Wigner–Ville frequency–time plot of the same fluctuating velocity time series shows that the peak frequency observed during a narrow wake remains constant and relatively high while the peak frequency observed during a wide wake remains constant and relatively low. The two frequency values are consistent with the peaks observed in the power spectra. Thus, during flopping, the high frequency observed in the velocity power spectra is always the peak frequency for a narrow wake and the low frequency observed in the velocity power spectra is always the peak frequency for a wide wake.

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**1. Introduction**

Many practical engineering applications can be modelled as the flow downstream of a plane array of cylinders placed normal to the flow. For infinitely long cylinders, the important geometrical parameters associated with the flow structure downstream of the cylinder arrays are the spacing between the top and bottom surfaces,  $s$ , of adjacent cylinders and the diameter of the cylinder,  $d$ .

The flow pattern about a cylinder is characterized, in part, by the magnitude of the base pressure coefficient ( $C_p = (P - P_\infty)/\frac{1}{2}\rho U_\infty^2$ ), which is always negative (hereafter, for simplicity, only the magnitude of  $C_p$  is used when comparing the flow patterns of different cylinder array geometries).

For a two-cylinder array, when the cylinder spacing-to-diameter ratio ( $s/d$ ) is set to equal to or greater than four, the near wake of each cylinder in the array is similar to that found downstream of an independent cylinder [cf. Bearman and Wadcock (1973)]. As the cylinder spacing is reduced to nearly zero ( $s/d \approx 0$ ) a single wake will be observed after a very short downstream distance. Previous studies of the transition from multiple independent wakes to a single wake have shown that, within a critical spacing ratio ( $0.1 < s/d < 1.3$ ), the average  $C_p$  of each cylinder varies in time and takes on two different values. Kim and Durbin (1988) describe this as the “flopping regime”. They suggest that flopping occurs spontaneously when the wake behind each cylinder alternates between a wide wake with a low magnitude  $C_p$  and a narrow wake with a high  $C_p$ .

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## 2. Background

Flopping behind two cylinders is studied by [Kim and Durbin \(1988\)](#). For their study,  $s/d$  is varied between 0.1 and 1, and the Reynolds number is varied between  $2.2 \times 10^3$  and  $6.2 \times 10^3$ . They present the first statistical analysis of the time interval for each period that the average  $C_p$  value remains relatively high combined with the time for each period that the average  $C_p$  value remains relatively low, and they find that the probability density function for the time intervals has a zero-event Poisson distribution. The average length of time between transitions decreases with increases in velocity.

[Eastop and Turner \(1982\)](#), with  $1.2 < s/d < 2.6$  and  $4.5 \times 10^4 < Re < 1.11 \times 10^5$ , observe the flow behavior downstream of a three-cylinder array while [Zdravkovich and Stonebanks \(1990\)](#), with  $1.1 < s/d < 1.75$  and  $3.7 \times 10^4 < Re < 7.4 \times 10^4$ , observe flopping with seven- and eleven-cylinder arrays placed normal and parallel to the freestream flow. Both studies present pressure distributions about the cylinders, velocity power spectra, and Strouhal numbers downstream of the center cylinder.

In an earlier study, [Bearman and Wadcock \(1973\)](#), for a two-cylinder array with the same  $s/d$  range and with a Reynolds number ( $Re$ ) of  $2.5 \times 10^4$  (based on cylinder diameter), also find that the average  $C_p$  of each cylinder has two different values. They experimentally observe velocity oscillations at higher frequencies when narrow wakes are present and velocity oscillations at lower frequencies when wide wakes are present. As have all past authors, they assume, but do not prove, that the peak frequency is constantly at the higher value when the narrow wake is present and constantly at the lower value when the wide wake is present.

The purpose of this study is to use time–frequency analysis during flopping to determine if: (i) both peak frequencies are always present and can be observed independently of whether the narrow or wide wake is present, or (ii) both peak frequencies are always present, but the higher frequency peak occurs more frequently for the narrow wake and the lower frequency occurs more frequently for the wide wake, or (iii) the peak frequency is constant and always at the higher value for the narrow wake, and it is constant and always at the lower value for the wide wake.

## 3. Facilities and approach

### 3.1. Pressure and hot-wire measurements

The wind tunnel, used for all experiments in the current study, has a test section with a length of 6.71 m, a cross-section of  $61 \times 91 \text{ cm}^2$ , and is preceded by a contraction section with a contraction ratio of 9.36.

The test cylinders used in this study are 1.27 cm in diameter, 0.30 m in length and are mounted at each end to identical 1.27 diameter, 0.62 m high rods that are attached to chemistry stands. A 0.01 mm diameter pressure port is located at the rear of each cylinder at the center plane of the array. The port intercepts a 0.48 cm diameter passage that is drilled along the center-line of the cylinder. The passage is blocked on one end of the cylinder and connected to a nipple on the other. This nipple is connected to a Setra (Model 339-1) differential pressure transducer. The output of the Setra transducer is connected to a Computer Boards Inc. (CBI) SSH-16 sample and hold board and a CBI CIO-AD16F 12-bit analogue-to-digital converter which are controlled by a PC. The pressure signal is sampled at 10 samples/s for a 3-h time period and filtered at 5 Hz.

The single hot-wire sensor that is used to obtain the frequency spectra is made by soldering a 2 mm long, 0.00508 mm diameter, Wollaston wire to a TSI-1210 sensor holder. The sensor holder is connected to a traverse that is located 45 cm downstream of the hot-wire sensor. The hot-wire sensor is connected to a TSI Model 1050 constant temperature anemometer and then to the same acquisition system used to obtain the pressure data. The data are collected at a sample rate of 2000 samples/s and filtered at 1000 Hz for a time period of 5 min.

### 3.2. Signal analysis

Techniques for producing frequency–time representations of a time-varying signal can be classified as either linear techniques or quadratic techniques ([Qian and Chen, 1996](#)). Specifically, although the Fourier transform is linear, since the power spectrum is created by squaring the results of the Fourier Transform, the Power Spectrum is quadratic. However, the Gabor and Wavelet transforms are used to create frequency–time distributions directly from the results of the Fourier transform and hence are linear. In contrast, the Wigner–Ville distribution is used to create frequency–time distributions directly from the Power Spectrum and is therefore a quadratic analysis technique. The Wigner–Ville distribution ([Jeong and Williams, 1992](#)) is based on a time-dependent power spectrum that is created by obtaining the Fourier transform of the auto-correlation of the time-varying signal. The two advantages of the Wigner–Ville

distribution are that only real numbers are computed and it has much better resolution than the Gabor and Wavelet transforms.

The Wigner–Ville distributions and corresponding frequency–time plots in this paper are generated using Matlab and the subroutine “TFDist” which is obtained from the library of Matlab routines at <http://www.stat.stanford.edu/~wavlab>.

#### 4. Results

For the two-cylinder array with  $s/d = 0.75$  and  $Re = 4.4 \times 10^3$ , the pressure behind each cylinder is measured as a function of time and is shown in Fig. 1. Each time the  $C_p$  value behind one of the cylinders rises, the  $C_p$  value behind the

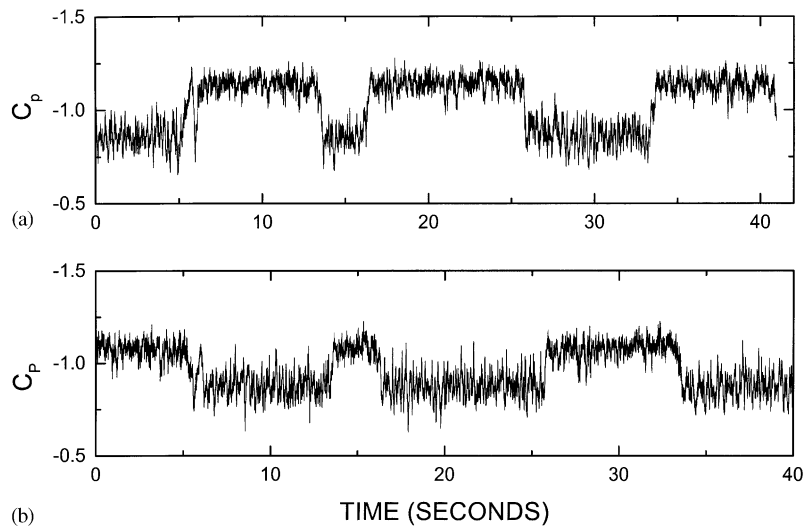


Fig. 1. Base pressure coefficient,  $C_p$ , as a function of time for a two-cylinder array with  $U = 6.8$  m/s and  $s/d = 0.75$ . (a) Corresponds to the top cylinder and (b) corresponds to the bottom cylinder.

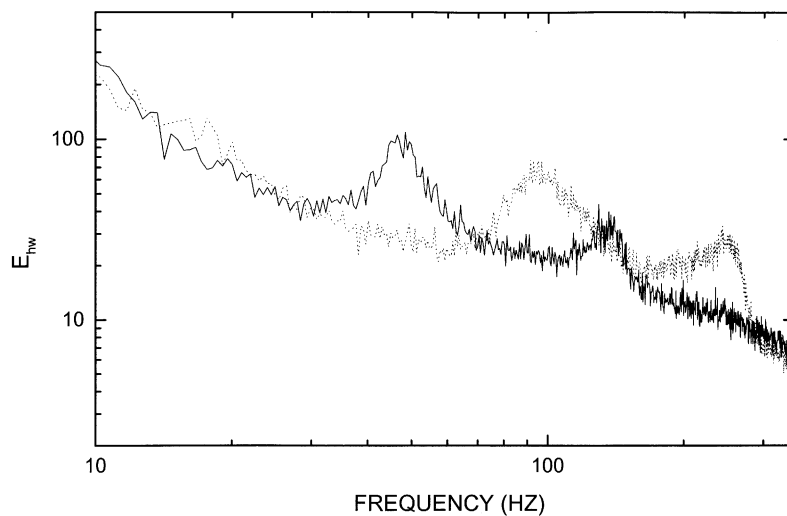


Fig. 2. Power spectra of an uncalibrated hot-wire signal,  $E_{hw}$ , for a two-cylinder array measured in the plane at  $s/2$  above the upper cylinder with  $s/d = 0.75$  at  $x/d = 3$ . The solid line corresponds to  $U = 5.2$  m/s and the dotted line corresponds to  $U = 10$  m/s.

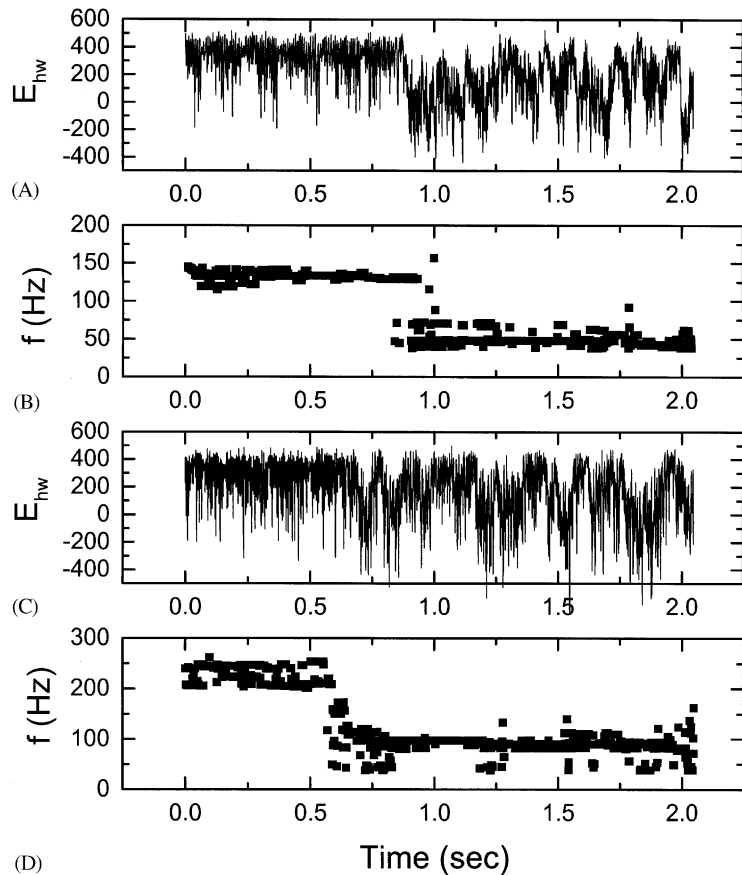


Fig. 3. Plots of an uncalibrated hot-wire signal,  $E_{hw}$ , and their corresponding frequency versus time representations. Fig. 3a and b represent the signal and frequency–time plot for  $U = 5.2$  m/s while Fig. 3c and d represent the signal and frequency–time plot for  $U = 10$  m/s.

other cylinder falls. This change in  $C_p$  is characteristic of flopping and shown by Kim and Durbin (1988) and Guillaume and LaRue (1999) to have a first-order Poisson distribution.

Power spectra obtained during flopping using the signal from an uncalibrated hot-wire which is placed at  $x/d = 3$  in the horizontal plane located at  $s/2$  above the top cylinder, and in the horizontal plane midway between the cylinders, is shown in Fig. 2. Dominant peaks are observed in the power spectra at 45 and 145 Hz for  $U = 5.2$  m/s and at 90 and 271 Hz for  $U = 10$  m/s.

Fig. 3a shows the uncalibrated hot-wire signal as a function of time for  $U = 5.2$  m/s. When flopping occurs at a time,  $t$ , of approximately 0.9 s, the characteristics of the signal change, and the wake downstream of the plate increases from a narrow shape to a wide shape (Guillaume and LaRue, 1999). The Wigner–Ville frequency–time plot that corresponds to the time series of Fig. 3a is shown in Fig. 3b. For about  $0 < t < 0.9$  s, the frequency remains relatively constant at about 145 Hz which is consistent with the high-frequency peak shown in the power spectra of Fig. 2 for  $U = 5.2$  m/s. For about  $0.9 < t < 2.1$  s, the frequency shown in Fig. 3b remains relatively constant at about 45 Hz which is consistent with the low-frequency peak shown in the power spectra of Fig. 2 for  $U = 5.2$  m/s.

Fig. 3c shows the uncalibrated hot-wire signal as a function of time for  $U = 10$  m/s. Flopping occurs at about  $t = 0.6$  s and the wake downstream of the plate changes from narrow to wide. In the corresponding Wigner–Ville frequency–time plot shown in Fig. 3d, the frequency for about  $0 < t < 0.6$  s remains relatively constant at about 271 Hz, and the frequency for about  $0 < t < 0.9$  s remains relatively constant at about 90 Hz. Again these two frequency values are consistent with the peaks observed in the power spectra of Fig. 2 with  $U = 10$  m/s.

## 5. Conclusion

Analysis of the frequency of the uncalibrated hot-wire signal obtained downstream of a two-cylinder array shows that the shedding frequencies observed for a narrow wake are constant and relatively high while the frequency observed for the wide wake are constant and relatively low. These frequencies directly correspond to the peaks in the velocity power spectra. Specifically, during flopping, the high frequency observed during a narrow wake is relatively constant and corresponds to the high peak observed in the velocity power spectra and the low frequency observed during a wide wake is relatively constant and corresponds to the lower peak in the velocity power spectra.

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